The Infinitesimal Calculus: How and Why it Was Imported into Europe

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Abstract:

It is well known that the “Taylor-series” expansion, that is the heart of the calculus, existed in India in widely distributed mathematics/ astronomy/ timekeeping (*jyotisa*) texts which preceded Newton and Leibniz by centuries. Why were these texts imported into Europe? These texts, and the accompanying precise sine values computed using the series expansions, were useful for the science then most critical to Europe—navigation—specifically for the problem of determining the three “ells”: latitude, loxodrome, and longitude. How were they imported? Jesuit records show that they sought out these texts as inputs to the Gregorian calendar reform, which, I point out, was needed to solve the latitude problem of European navigation. The Jesuits were equipped with knowledge of both the local language and the mathematics and astronomy needed to understand these texts, and they needed these texts also to understand local customs, and how dates of traditional festivals were fixed using the local calendar (*pancânga*). How the mathematics in these texts subsequently diffused into Europe (e.g. through clearinghouses like Mersenne, and the works of Cavalieri, Fermat, Pascal, Wallis, Gregory etc.) is another story.

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Extended summary:

The calculus has played a key role in the development of the sciences, starting from the "Newtonian Revolution". According to the "standard" story, the calculus was invented independently by Leibniz and Newton. This story of indigenous development, *ab initio*, is now beginning to totter, like the story of the "Copernican Revolution". The English-speaking world has known for over one and a half centuries that "Taylor" series expansions for sine, cosine and arctangent functions were found in Indian mathematics/astronomy/timekeeping (*jyotisa*) texts, and specifically in the works of Madhava, Neelkantha (*Tantrasangraha*, 1501CE), Jyeshtadeva (*Yuktibhâsâ*, c. 1530 CE) etc. No one else, however, has so far studied the connection of these Indian developments to European mathematics.

The relation is provided by the requirements of the European navigational problem, the foremost problem of the time in Europe. Columbus and Vasco da Gama used dead reckoning and were ignorant of celestial navigation. Navigation, however, was both strategically and economically the key to the prosperity of Europe of that time. Accordingly, various European governments acknowledged their ignorance of navigation, while announcing huge rewards to anyone who developed an appropriate technique of navigation. These rewards spread over time from the appointment of Nunes as Royal Cosmographer in 1529, to the Spanish government's prize of 1567 through its revised prize of 1598, the Dutch prize of 1636, Mazarin's prize to Morin of 1645, the French offer (through Colbert) of 1666, and the British prize legislated in 1711. Many key scientists of the time (Huygens, Galileo, etc.) were involved in these efforts: the navigational problem was the specific objective of the French Royal Academy, and a key concern for starting the British Royal Society.

Prior to the clock technology of the 18th century, attacks on the navigational problem in the 16th and 17th c. focussed on mathematics and astronomy, which were (correctly) believed to hold the key to celestial navigation, and it was widely (and correctly) believed by navigational theorists and mathematicians (e.g. by Stevin and Mersenne) that this knowledge was to be found in ancient mathematical and astronomical or time-keeping (*jyotisa*) texts of the east. Though the longitude problem has recently been highlighted, this was preceded by a latitude problem, and the problem of loxodromes.

The solution of the latitude problem required a reformed calendar: the European calendar was off by 10 days, and this led to large inaccuracies (more than 3 degrees) in calculating latitude from measurement of solar altitude at noon, using e.g. the method described in the *Laghu Bhâskarîya* of Bhaskara I. However, reforming the calendar required a change in the dates of the equinoxes, hence a change in the date of Easter, and this was authorised by the Council of Trent in 1545. This period saw the rise of the Jesuits. Clavius studied in Coimbra under the mathematician, astronomer and navigational theorist Pedro Nunes, and Clavius subsequently reformed the Jesuit mathematical syllabus at the Collegio Romano. Clavius also headed the committee which authored

the Gregorian Calendar Reform of 1582, and remained in correspondence with his teacher Nunes during this period.

Jesuits, like Matteo Ricci, who trained in mathematics and astronomy, under Clavius' new syllabus [Ricci also visited Coimbra and learnt navigation], were sent to India. In a 1581 letter, Ricci explicitly acknowledged that he was trying to understand local methods of timekeeping from "an intelligent Brahmin or an honest Moor", in the vicinity of Cochin, which was, then, the key centre for mathematics and astronomy, since the Vijaynagar empire had sheltered it from the continuous onslaughts of raiders from the north. Language was hardly a problem, for the Jesuits had established a substantial presence in India, had a college in Cochin, and had even started printing presses in local languages, like Malayalam and Tamil by the 1570's.

Though the difficulties with the calendar were settled by the Gregorian Reform, there remained the problem of precise sine values which were also needed for calculating latitude from a local observation of solar altitude at noon. Sine tables were used also to calculate loxodromes, which were the focus of efforts of navigational theorists like Nunes, Mercator etc. (The problem of calculating loxodromes is exactly the problem of the fundamental theorem of calculus.) Hence, Nunes, Stevin, Clavius etc. were greatly concerned with accurate sine values, and each of them published lengthy sine tables. Not only does the very word 'sine' derive from a Latin mis-translation of the term *jîbâ* used in Arabic for the Indian *jîvâ*, but Clavius, for example, used the Indian definition of the sine, and Stevin mentions Aryabhata's value of π. Madhava's sine table, which extended Arayabhata's sine table, using the series expansion of the sine function, were then the most accurate sine values available, and the coefficients needed to calculate these values, in a numerically efficient way, were encapsulated in a couple of verses in various widely distributed mathematics/astronomy/timekeeping (*jyotisa*) texts, including the *Karanapaddhati*.

Sine values could also be used to determine longitude.* But, Europeans encountered difficulties in using these precise sine value for determining longitude, as in Indo-Arabic navigational techniques or in the *Laghu Bhâskarîya*, because this technique of longitude determination also required an accurate estimate of the size of the earth, and Columbus had underestimated the size of the earth to facilitate funding for his project of sailing West. Columbus' incorrect estimate was corrected, in Europe, only towards the end of the 17th c. CE. Even so, the Indo-Arabic navigational technique required *calculation*, while Europeans lacked the ability to calculate, since algorismus texts had only recently triumphed over abacus texts, and the European tradition of mathematics was "spiritual"

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* C. K. Raju, "Kamâl or Râpalagâi". Paper presented at the Indo-Portuguese Conference on History, Indian National Science Academy, New Delhi, 1998. (To appear in Proc.) The Kamâl was the navigational instrument used by the Indian pilot/ *muâlîm/ mâlmî* / Malemo who brought Vasco da Gama from Africa to India in 1498.

and “formal” rather than practical, as Clavius had acknowledged in the 16th c. and as Swift (*Gulliver’s Travels*) had satirized in the 17th c. Finally, the transmission of the calculus, based on a foreign epistemology,[*] led to an epistemological discontinuity that could be resolved in Europe only in the 19th c., with the development of real numbers and mathematical analysis, until which time the calculus was viewed with a suspicion similar to the suspicion that had earlier been directed towards zero, for a similar prolonged period of a few centuries. This led to the development of the chronometer, an appliance that could be mechanically used without application of the mind.

About the author:

C. K. Raju holds a Ph.D. from the Indian Statistical Institute. He taught mathematics for several years before playing a lead role in the C-DAC team which built Param: India’s first parallel supercomputer. His earlier book *Time: Towards a Consistent Theory* (Kluwer Academic, 1994) set out a new physics with a tilt in the arrow of time. His more recent work, *The Eleven Pictures of Time: The Physics, Philosophy, and Politics of Time Beliefs* (Sage, 2002) deals with time at the interface of science and religion. He has been a Fellow of the Indian Institute of Advanced Study and is Professor of Computer Science. He is an editor of the *Journal of Indian Council of Philosophical Research*, and an Editorial Fellow of the Centre for Studies in Civilizations, bringing out a three part history of science and technology in modern India for the Project of History of Indian Science, Philosophy, and Culture. The first of these books, entitled, *Cultural Foundations of Mathematics: Prolegomenon to a History of Science in Modern India*, is scheduled for next year. He also coordinates an Indian National Science Academy project on “Madhava and the Origin of the Differential Calculus” and is an Affiliated Fellow of the Nehru Memorial Museum and Library.

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[*] C. K. Raju, “Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the Yuktibhâsâ”, *Philosophy East and West* **51** (3) 2001,325–362. C. K. Raju, “How Should ‘Euclidean’ Geometry be Taught?” in G. Nagarjuna (ed) *History of Science: Implications for Science Education*, Homi Bhabha Centre, TIFR, Bombay, 2002, 241–260.